# Short Papers

## Local-Oscillator Noise Cancellation in the Subharmonically Pumped Down-Converter

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***Abstract*—The noise power at the IF output of a superheterodyne mixer which is caused by local-oscillator noise can be significantly reduced by using the recently developed subharmonically pumped down-converter. In many cases this reduction is so large that even noisy sources, such as IMPATT oscillators, can be used to pump low-noise mixers without causing significant degradation of noise figure.**

### I. Introduction

A new type of mixer called a subharmonically pumped down-converter has recently been developed [1]–[4]. The mixer is particularly attractive for use at millimeter-wave frequencies because it can be pumped at one-half the frequency required for a conventional mixer, yet shows negligible increase in conversion loss.

An important property of this mixer, first discussed by Cohn *et al.* [1], [4], is the strong attenuation of down-converted local-oscillator noise available at the IF output. The amount of attenuation is determined by the noise spectrum of the local oscillator. In many cases it is sufficiently large so that the noise power at IF due to a noisy local oscillator, such as an IMPATT diode, is negligible. In this short paper we present a detailed discussion of this phenomenon, and report measurements on a 60-GHz mixer (local oscillator at 30 GHz), which show a noise reduction of at least 19 dB relative to the noise expected from a conventional, single-ended mixer using the same local oscillator.

Usually, the contribution of local-oscillator noise to IF noise power output can be reduced by using a balanced mixer, but at millimeter-wave frequencies the balanced mixer presents cumbersome mechanical problems. The new mixer, however, is relatively simple to build. We conclude that this down-converter provides an attractive solution to the problem of using noisy sources as local oscillators at millimeter-wave frequencies.

### II. Response of Mixers to Local-Oscillator Noise

It is well known that conventional single-ended mixers are susceptible to degradation by local-oscillator noise [5], [6]. The cause of the problem is illustrated in Fig. 1. We assume in Fig. 1, and throughout this paper, that $\omega_{IF} \ll \omega_{LO}$. Noise components in the local-oscillator sidebands at $\omega_{LO} \pm \omega_{IF}$ beat with the local oscillator to produce noise power at IF. These noise sidebands are of nearly equal amplitude, resulting in a signal-to-noise ratio of

$$\mathrm{SNR} = \frac{\frac{1}{2}V_s^2}{2n^2(\omega_{LO} + \omega_{IF}) \cdot B} \tag{1}$$

where $V_s$ is the peak input signal voltage, $n^2(\omega)$ is the mean-squared noise voltage per unit bandwidth at $\omega$, and $B$ is the IF bandwidth.

The subharmonically pumped down-converter provides a way to reduce the signal-to-noise degradation which is caused by local-oscillator noise. This property is directly related to the fact that the $I$–$V$ curve of the nonlinear element in the mixer is an antisymmetric function of voltage. The relationship between symmetry and noise reduction is illuminated by the following discussion.

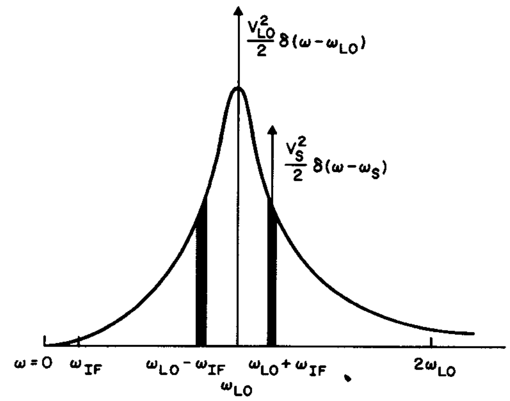


Fig. 1. Spectrum of a noisy local oscillator, plus input signal at $\omega_{LO} + \omega_{IF}$. The size of $\omega_{IF}$ relative to $\omega_{LO}$ has been exaggerated for clarity. The indicated noise sidebands are appropriate for a conventional mixer.

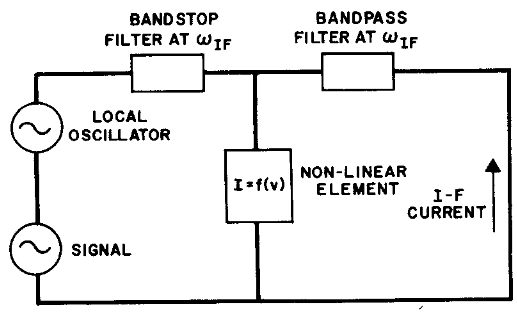


Fig. 2. A simple mixer.

Consider the mixer shown in Fig. 2. The voltages from the local oscillator and signal are impressed across the nonlinear element. The filters separate the current flowing at $\omega_{IF}$ from the other components of diode current. Assume the current through the nonlinear element, $I$, to be a function only of the voltage across it, $v$:

$$I = f(v). \tag{2}$$

For a conventional mixer the function $f$ is the sum of symmetric and antisymmetric parts, $f_s(v)$ and $f_a(v)$

$$I(v) = f_s(v) + f_a(v) \tag{3}$$

where

$$f_s(v) = [f(v) + f(-v)]/2$$

and

$$f_a(v) = [f(v) - f(-v)]/2.$$

For this calculation we will be interested in local-oscillator noise components at frequencies far from the carrier, i.e., in the wings of the local-oscillator spectrum. In this case we can represent the local oscillator as the sum of a pure sinusoid of amplitude $V_{LO}$, plus a small noise voltage $n(t)$ [7]. The total

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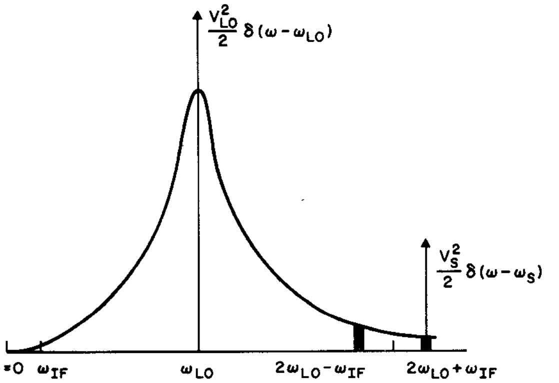


Fig. 3. Spectrum of a noisy local oscillator, plus input signal at $2\omega_{LO} + \omega_{IF}$. The indicated noise sidebands are appropriate for an antisymmetric mixer.

voltage across the nonlinear element is then

$$v(t) = V_{LO} \sin \omega_{LO}t + n(t) + V_s \sin \omega_s t \tag{4}$$

where $V_s$ is the signal amplitude, and $\omega_s$ is the signal frequency. For signal and noise voltages that are small compared with $V_{LO}$ we can develop the current in a Taylor series

$$I = f_s(V_{LO} \sin \omega_{LO}t) + f_a(V_{LO} \sin \omega_{LO}t) + \{V_s \sin \omega_s t + n(t)\} \cdot \left\{\frac{df_s(v)}{dv} + \frac{df_a(v)}{dv}\right\}_{v=V_{LO}\sin\omega_{LO}t} \tag{5}$$

From the final term in (5) we can find the components of signal and noise currents flowing at $\omega_{IF}$. Since $f_s(v)$ is symmetric in $v$, $df_s(v)/dv$ is antisymmetric. It is well known that if a function of a variable with period $T$ is antisymmetric in that variable, the Fourier series representation of that function contains only terms at frequencies $\omega$, $3\omega$, $5\omega, \cdots$, where $\omega = 2\pi/T$. Therefore the Fourier expansion of

$$\left(\frac{df_s(v)}{dv}\right)_{v=V_{LO}\sin\omega_{LO}t}$$

contains only odd harmonics of $\omega_{LO}$: $\omega_{LO}$, $3\omega_{LO}$, $5\omega_{LO}$, etc. The frequencies of the time-varying terms in the expansion of

$$\left(\frac{df_a(v)}{dv}\right)_{v=V_{LO}\sin\omega_{LO}t}$$

on the other hand, will be only the even harmonics of $\omega_{LO}$: $2\omega_{LO}$, $4\omega_{LO}$, etc. Thus in a conventional mixer the signal current at IF is caused by the "beat" between the signal at $\omega_s = \omega_{LO} \pm \omega_{IF}$ and the component of $df_s(v)/dv$ at $\omega_{LO}$. There is no IF signal contribution from $df_a(v)/dv$ because it has no component at $\omega_{LO}$. The IF signal-to-noise ratio is determined primarily by the relative strengths of signal and local-oscillator noise at $\omega_{LO} \pm \omega_{IF}$, as shown in Fig. 1.

Now consider a mixer where $f(v)$ is purely antisymmetric, so that $f_s = 0$. In this case the IF signal current is the beat between the component of $df_a(v)/dv$ at $2\omega_{LO}$ and the signal at $2\omega_{LO} \pm \omega_{IF}$, as shown in Fig. 3. The noise components at $\omega_{LO} \pm \omega_{IF}$ cannot contribute to the IF current. Rather, it is the components at $2\omega_{LO} \pm \omega_{IF}$ that are important. Since local-oscillator noise tends to drop off rapidly away from the carrier [7], the relevant oscillator noise components are much smaller at $2\omega_{LO} \pm \omega_{IF}$ than at $\omega_{LO} \pm \omega_{IF}$. Hence, for a given input signal, the signal-to-noise ratio will be much higher for the purely antisymmetric mixer than for the conventional mixer.

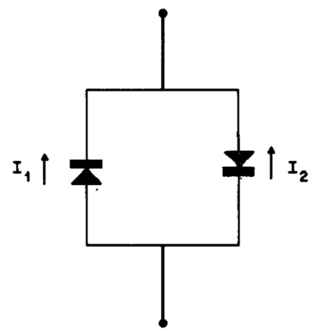


Fig. 4. The nonlinear element of a subharmonically pumped down-converter.

In the following section we will study an example of a mixer which displays these desirable symmetry properties.

## III. The Subharmonically Pumped Down-Converter

The subharmonically pumped down-converter is a mixer in which the nonlinear element is a pair of antiparallel diodes, as shown in Fig. 4. The total current through the diode pair is given by the algebraic sum of the separate currents. For ideal diodes with negligible parasitic resistance and capacitance

$$I = I_1 + I_2 = I_s(e^{\alpha v} - 1) - I_s(e^{-\alpha v} - 1) = 2I_s \sinh \alpha v \tag{6}$$

where $I_s$ is the diode reverse saturation current and $\alpha = q/nkT \approx (25 \text{ mV})^{-1}$ [3]. From (6) the antisymmetry of the $I$–$V$ characteristic is clear. For the rest of this short paper we will refer to the subharmonically pumped down-converter as the hyperbolic-sine-law mixer.

To study the response of the hyberbolic-sine-law mixer to local-oscillator noise, we assume, as before, that the total applied voltage is the sum of the local oscillator, signal, and noise voltages. The total diode current is

$$I = 2I_s \sinh \alpha\{V_{LO} \sin \omega_{LO}t + V_s \sin \omega_s t + n(t)\}. \tag{7}$$

For small signal and noise, i.e.,

$$\alpha\{V_s \sin \omega_s t + n(t)\} \ll 1 \tag{8}$$

we find the current to be

$$I \simeq 2\alpha\{V_s \sin \omega_s t + n(t)\}I_s \cosh (\alpha V_{LO} \sin \omega_{LO}t) + 2I_s \sinh (\alpha V_{LO} \sin \omega_{LO}t). \tag{9}$$

All the components of current flowing at $\omega_{IF}$ are contained in the first term on the right-hand side of (9). Therefore we neglect the second term and expand the hyperbolic cosine in a Fourier series

$$I \simeq 2I_s\alpha\{V_s \sin \omega_s t + n(t)\} \cdot \{I_0(\alpha V_{LO}) + 2 \sum_{k=1}^{\infty} (-1)^k I_{2k}(\alpha V_{LO}) \cdot \cos 2k\omega_{LO}t\} \tag{10}$$

where the $I_{2k}$ are the hyperbolic Bessel functions [8]. The argument of the functions, $\alpha V_{LO}$, is approximately 24, because $V_{LO}$ must be $\sim$0.6 V in order to get appreciable current to flow through the mixer diodes [3]. For this argument the first few terms of the $I_{2k}$ form a slowly decreasing sequence. Thus the hyperbolic-sine-law mixer is capable of high-order harmonic mixing [3], [4]. In normal operation, however, the mixer is used to receive signals at $\omega_s = 2\omega_{LO} \pm \omega_{IF}$. If we assume, as we did in Section II, that local-oscillator noise falls off rapidly away from the carrier, then the dominant noise contribution is made by noise components near $2\omega_{LO} \pm \omega_{IF}$, as shown in Fig. 3. Since these are nearly equal for $\omega_{IF} \ll \omega_{LO}$, we have

$$\mathrm{SNR} = \frac{\frac{1}{2}V_s^2}{2n^2(2\omega_{LO} + \omega_{IF}) \cdot B}. \tag{11}$$

Compared with the performance of the conventional mixer given by (1), the hyperbolic-sine-law mixer, with "effective" local oscillator at $2\omega_{LO}$, offers a noise improvement of

$$\eta = 10 \log \frac{n^2(\omega_{LO} + \omega_{IF})}{n^2(2\omega_{LO} + \omega_{IF})} \text{ dB}. \tag{12}$$

This is the principal result of our analysis. It is valid provided that oscillator noise at higher order even harmonics is negliglble compared with that at $2\omega_{LO}$.

It should be remarked that the antisymmetry of the $I$–$V$ curve for a mixer with two identical antiparallel diodes is unaffected by the presence of diode series resistance. Since the argument following (5) in Section II shows that noise suppression is related to antisymmetry and not to the details of the shape of the $I$–$V$ curve, the noise improvement given by (12) also applies to diodes with series resistance.

An estimate of the noise improvement available with the hyperbolic-sine-law mixer requires a model for the noise spectrum of the local oscillator. However, it is often difficult to know accurately the behavior of the oscillator circuit elements at frequencies near $2\omega_{LO}$. To make an order-of-magnitude estimate of oscillator noise, let us assume that the oscillator is represented by a frequency-independent negative conductance in parallel with a singly tuned resonant circuit. For such an oscillator the noise spectral density far from the carrier has the same frequency dependence as the response of an $RLC$ circuit to white noise [7][1]

$$n^2(\omega) \propto \left[S^2 + Q^2\left(\frac{\omega}{\omega_{LO}} - \frac{\omega_{LO}}{\omega}\right)^2\right]^{-1} \tag{13}$$

where $S$ is the saturation parameter of the oscillator [7], and $Q$ is its external $Q$. In those cases where $\omega_{IF} \gtrsim S\omega_{LO}/Q$, so that we need not be concerned with noise components near the resonant peak, (12) and (13) can be combined to yield a simple result

$$\eta' = 20 \log (3\omega_{LO}/4\omega_{IF}). \tag{14}$$

Equation (14) is useful as a guide to the noise reduction expected with the hyperbolic-sine-law mixer.

## IV. Experimental Results

Measurements of the noise performance of a 60-GHz hyperbolic-sine-law mixer with IF at 1.4 GHz show clearly the reduction of the IF noise contribution from the local oscillator. For these experiments the required local-oscillator power of ~5 mW was supplied by a 30-GHz microstrip IMPATT oscillator [10]. The IMPATT local oscillator caused the mixer noise figure to increase not more than 1 dB over the 10-dB value measured using a klystron local oscillator with negligible noise [11]. Thus we have

$$10 \log \left(\frac{N_{\sinh}}{kT} + 10\right) \leq 11 \text{ dB} \tag{15}$$

[1] Reference [7, eq. (41)] is an approximate expression valid only near $\omega = \omega_{LO}$. See [9].

where $N_{\sinh}$ is the contribution to IF noise from the local oscillator referred to the mixer input, and $kT \simeq 4 \times 10^{-18}$ mW/Hz. Therefore,

$$N_{\sinh} \leq 10^{-17} \text{ mW/Hz}. \tag{16}$$

Let us compare this with the effective noise that would be expected using this same local oscillator with a conventional mixer (IF = 1.4 GHz) to receive a signal near 30 GHz. At 5-mW power level, the single-sideband noise power spectral density of the local oscillator is measured to be $4 \times 10^{-16}$ mW/Hz at 1.4 GHz away from the carrier. From (1) we see that the noise bands at $\omega_{LO} \pm \omega_{IF}$ both contribute to IF noise power, giving an effective noise density, referred to the mixer input, of

$$N_{conv} = 8 \times 10^{-16} \text{ mW/Hz}. \tag{17}$$

Comparing (16) and (17), we find the ratio of IF noise powers for the conventional and hyperbolic-sine-law mixers to be

$$\frac{N_{conv}}{N_{\sinh}} \geq 19 \text{ dB}. \tag{18}$$

That is, we have observed a noise reduction of at least 19 dB. The value of $\eta'$ calculated from (14) is 24 dB.

The noise-reduction capability of the hyperbolic-sine-law mixer can be degraded if the diodes are not identical. This destroys the antisymmetry of the mixer and allows local-oscillator noise at $\omega_{LO} \pm \omega_{IF}$ to contribute to the IF noise current. If we call this contribution $P_1$, and let $P_2$ be the IF contribution due to local-oscillator noise at $2\omega_{LO} \pm \omega_{IF}$, then (see Appendix)

$$\frac{P_1}{P_2} \approx \left[2\frac{I_{dc}}{I_{LO}} \cdot \frac{\omega_{LO}}{\omega_{IF}}\right]^2 \tag{19}$$

where $I_{dc}$ is the net dc current flowing through the diodes ($I_{dc} = 0$ for identical diodes), and $I_{LO}$ is the amplitude of the diode current flowing at $\omega_{LO}$. In the mixer described in this section $I_{dc}$ is ~30 $\mu$A for 5-mW local-oscillator power. Since the voltage across the diodes is ~0.6 V [3], and the power absorbed is ~5 mW, the current flowing through them, $I_{LO}$, is of order 10 mA. Substituting these values of $I_{dc}$ and $I_{LO}$ into (19), we find

$$\frac{P_1}{P_2} \approx 0.02. \tag{20}$$

Thus, for the mixer described here, diode imbalance causes a negligible increase in total mixer noise.

## V. Reception of Angle-Modulated Signals

Phase noise (FM noise) in the local oscillator causes random phase fluctuations in the IF output signal, which can produce radio system degradation if the signal is angle modulated. However, even though the hyperbolic-sine-law mixer doubles the rms phase fluctuations of the local oscillator [see (10)], its output phase noise is not significantly greater than that of a conventional mixer receiving the same signal. This is because the local oscillator for the hyperbolic-sine-law mixer, running at about half the frequency of the local oscillator for the conventional mixer, will show about half the rms phase fluctuations, provided the two oscillators are of similar design [12]. The doubling of these fluctuations by the hyperbolic-sine-law mixer results in IF fluctuations roughly equal to those of a conventional mixer.

Experimentally, we observed that an IMPATT local oscillator with phase noise of ~400 Hz/$\sqrt{\text{kHz}}$ produces phase noise of

$\sim 700\ \text{Hz}/\sqrt{\text{kHz}}$ at the IF output of a hyperbolic-sine-law mixer. Considering our probable error of 10–20 percent on each of those measurements, the data are consistent with the hypothesized doubling of local oscillator FM noise by the hyperbolic-sine-law mixer.

## Appendix
### Effect of Diode Imbalance on Mixer Noise

An expression relating diode imbalance to hyperbolic-sine-law mixer noise can be derived in a straightforward fashion. Assume that the diodes have identical saturation current but slightly different $\alpha$ parameters. The diode current is given by

$$I(v) = I_s(e^{\alpha v} - e^{-(\alpha + \delta\alpha)v}) \tag{A-1}$$

where $\delta\alpha/\alpha \ll 1$. If $V_{\text{LO}}$ is much greater than the signal and noise voltages, we can obtain an expansion of (A-1) similar to (10). From this we find the components of the total current at dc (which is due to diode imbalance) and $\omega_{\text{LO}}$, as well as the contributions of local-oscillator noise at $\omega_{\text{LO}} \pm \omega_{\text{IF}}$ and $2\omega_{\text{LO}} \pm \omega_{\text{IF}}$. Using the notation of Sections III and IV we have

$$\frac{I_{\text{dc}}}{I_{\text{LO}}} = \frac{\delta\alpha}{\alpha} \cdot \frac{\alpha V_{\text{LO}}}{8} \tag{A-2}$$

and

$$\frac{P_1}{P_2} = \left[\frac{(\delta\alpha/\alpha) \cdot \alpha V_{\text{LO}} \cdot I_0(\alpha V_{\text{LO}})}{4 I_2(\alpha V_{\text{LO}})}\right]^2 \cdot \frac{n^2(\omega_{\text{LO}} \pm \omega_{\text{IF}})}{n^2(2\omega_{\text{LO}} \pm \omega_{\text{IF}})}. \tag{A-3}$$

Equations (A-2) and (A-3) can be combined and simplified by noting that $I_0(\alpha V_{\text{LO}}) \approx I_2(\alpha V_{\text{LO}})$, and assuming that $n^2(\omega)$ falls off roughly like $(\omega_{\text{LO}} - \omega)^{-2}$ away from the carrier. Then

$$\frac{P_1}{P_2} \simeq \left[2\frac{I_{\text{dc}}}{I_{\text{LO}}} \cdot \frac{\omega_{\text{LO}}}{\omega_{\text{IF}}}\right]^2. \tag{A-4}$$

Equation (A-4) shows that when $\omega_{\text{IF}} \ll \omega_{\text{LO}}$, extremely good diode balance is required to ensure $P_1 \ll P_2$.

A calculation assuming identical $\alpha$ parameters but slightly different values of the saturation current also yields the result given in (A-4).

## Acknowledgment

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# The Ridged-Waveguide-Cavity Gunn Oscillator for Wide-Band Tuning

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***Abstract*—A method of construction of the Gunn oscillator for wide-band frequency tuning has been developed using the ridged-waveguide cavity. The ridged waveguide can be designed to provide the dominant mode with a wide bandwidth, and also to provide the higher order cutoff modes whose resonance is a limiting factor of the tuning range for oscillators of conventional design with reduced stored energies to permit wide-band tuning. A prototype oscillator having a packaged *X*-band diode demonstrated an 8–18-GHz tuning range.**

## Introduction

In the course of investigating a transmission line that would be suitable for use with semiconductor devices at high microwave frequencies (from the *X*-band to the millimeter wavelengths where control of parasitics of the circuit and the package becomes all the more important for wide-band applications) the ridged-waveguide-cavity Gunn oscillator was developed. The use of the Gunn diode offers a convenient means of testing properties of the microwave circuit over a very wide frequency range because of its capability for wide-band operation [1], [2]. This short paper presents the structure and the performance characteristics of the ridged-waveguide-cavity Gunn oscillator operating at frequencies from 8 to 18 GHz.

## Theory

Fig. 1 illustrates the oscillator structure where the ridges extend uniformly all the way to the shorting plunger. The field in the waveguide excited by the packaged diode may be represented by a sum of the normal modes of the ridged waveguide [3], of which only the dominant $TE_{10}$ mode propagates while all other modes evanesce at frequencies of interest. The bandwidth of $TE_{10}$ single-mode transmission as well as its characteristic impedance is controlled by the combination of $a$, $b$, $w$, and $g$, referring to the figure. The amplitudes of the evanescent modes are kept small by making the difference small between the cross section in the plane at the diode center and that of the waveguide. In addition, the diode placed at the center of the symmetric waveguide decouples from a certain class of asymmetric modes. The evanescent modes are made to evanesce quickly by choosing $a$ and $b$ small, thereby reducing the reactive energies associated with these modes, so long as they are consistent with the bandwidth and impedance requirements. The geometry of the ridged waveguide has the needed flexibility to meet these requirements.

## Experiment

Several units of the oscillator were built and tested using packaged *X*-band Gunn diodes and ridged waveguides having various cross sectional shapes and dimensions. A typical tuning

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